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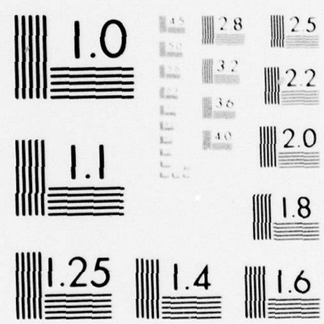
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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

ON A DISCRETE VORTEX SCHEME FOR A FINITE SPAN WING

By

N. F. Vorob'yev

FOREIGN TECHNOLOGY DIVISION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

ON A DISCRETE VORTEX SCHEME FOR A FINITE SPAN WING

N. F. Vorob'yev

This article examines the task of the flow around the lifting surface of a wing by a nonviscous, incompressible flow. The wing surface itself is replaced by a vortical surface S, and the veil of the vortices which descend from the rear and, in the general case, also from the side and leading edges of the wing is a vortical surface Σ which consists of vortices whose axes with steady motion are directed along the lines of current. The vortical densities $\bar{\rho}$ on the surfaces S and Σ are determined from the condition of non-passage. Furthermore, the conditions of descent should be satisfied on the edges of the wing. The presence of a vortical veil descending from the wing edges provides the opportunity to ensure the condition of limitedness of velocity on the edges from which the vortical veil descends [1-3]. In the case of a linear scheme where the vortical veil descends only from the trailing edge, on the leading and lateral edges of the wing, as is known, the rate of inflow which is determined within the framework of an ideal fluid is infinitely great. In a nonlinear case, where the shape of the surface is unknown, the solution of integral equations for wings of complex form in a plane is difficult [1, 2].

There is a method for calculating the aerodynamic characteristics of a flat wing of arbitrary shape in a plane where the

vortical layer which simulates the wing surface is replaced by a system of discrete vortices whose intensity is determined from the nonpassage condition [4]. The vortical veil outside the wing is also simulated by discrete vortical lines which are a continuation of the vortices on the surface of the wing itself. Each of the vortical lines outside the wing is presented as consisting of straight segments which receive the direction of the velocity in the corresponding point in space. The position of the vortical lines outside the wing is determined by the method of successive approximations in the course of calculation. Solution of the problem of flow around a wing of finite span in accordance with the scheme of discrete vortices is reduced to the solution of a system of algabraic equations; this method is convenient for realization on a computer for wings of arbitrary shape in a plane. The wing can be represented by a system of discrete vortices and, in this case, when it is nonplaner. In the discrete scheme, a finite span wing remains an open question concerning convergence with an increase in the number of discrete vortices which replace the wing and concerning the satisfaction of conditions for descent on the wing edges.

In this work it is shown that with the appropriate selection of discrete vortices which replace the wing surface and of points in which the nonpassage conditions are satisfied, with an increase in the number of vortices the algabraic sums through which the velocities induced on the wing surface by the discrete vortices are presented are transformed to integrals whose convergence is proven and the introduction of additional vortices close to the edges ensures the satisfaction of the descent condition.

A wing of arbitrary form in a plane in the general case is some smooth surface S which can be covered by an orthogonal grid of curvilinear coordinates connected with the wing surface. The coordinate system is selected in such a way that the line ξ = const connects the leading and trailing edges of the wing (Fig. 1). A discrete coordinate grid ξ = const, ζ = const, which divides the

wing into rectangles with sides $\Delta\xi_i$, $2\Delta\zeta_j$ is plotted on the surface. One of the coordinate lines passes through the point $N(\xi_i, \zeta_j)$ and is the coordinate line of the grid $\xi = \xi_i$. Two other coordinate lines of the grid which are closest to the point $N(\xi_i, \zeta_j)$ and orthogonal to the line $\xi = \xi_i$ are selected in such a manner that the point $N(\xi_i, \zeta_j)$ is the middle of the side of the coordinate grid: this line $\zeta = \zeta_j - \Delta\zeta_j$, $\zeta = \zeta_j + \Delta\zeta_j$.

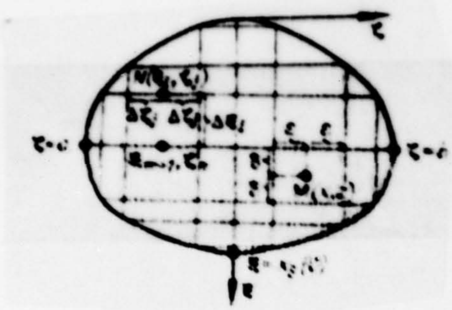

Fig. 1.

The selection of direction of the vortical lines which replace the wing can be arbitrary. These vortices do not follow the laws of behavior of a vortical line in the flow of an ideal liquid. It is assumed that the segment of a coordinate line whose middle is point $N(\xi_i, \zeta_j)$ is a segment of a vortical line of constant intensity. Points $N(\xi_i, \zeta_j)$ through which Π-shaped vortices pass can be numbered by lines and columns: each point is assigned a number (m, n). At points $(\xi_i, \zeta_j - \Delta\zeta_j)$, $(\xi_i, \zeta_j + \Delta\zeta_j)$ a vortical line undergoes a fracture and continues along coordinate lines $\zeta = \zeta_j - \Delta\zeta_j$, $\zeta = \zeta_j + \Delta\zeta_j$ to the trailing edge of the wing.

The next vortical line of constant intensity $\Delta\Gamma_{m+1,\,n}$ is located behind the Π-shaped vortex of intensity $\Delta\Gamma_{ij}$ on the wing on the segment of the coordinate line $\xi = \xi_i + \Delta\xi_i = \xi_{m+1}$, the middle of which is point $\xi_i + \Delta\xi_i$, ζ_j. At points $(\xi_i + \Delta\xi_i, \zeta_j - \Delta\zeta_j)$, $(\xi_i + \Delta\xi_i, \zeta_j + \Delta\zeta_j)$ this vortical line also experiences a fracture and continues along the coordinate lines $\zeta = \zeta_j - \Delta\zeta_j$, $\zeta = \zeta_j + \Delta\zeta_j$ to the trailing edge of the wing.

Thus, the entire wing S is covered by a system of rectangular discrete Π-shaped vortices connected with the wing. The Π-shaped vortices which are located on the wing S as well as some additional number of discrete vortices which will be introduced later to satisfy the conditions of descent on the edges of the wing descend from the wing and continue outside the wing to infinity, simulating the vortical veil Σ. The vortical lines which represent the vortical veil Σ consist of finite straight segments which are aligned in accordance with the direction of the velocity in the corresponding points outside the wing [4]. The intensity of the vortices is determined from the condition of non-passage on the wing surface and the conditions of descent on the edges of the wing. A special feature which is connected with the sections of joined vortices appears with a decrease in the size of the cells in the expression of the velocity induced in the points of the wing surface. The regular part of the expression of velocity which is induced in the points of the wing surface S is connected with sections of free vortices which are the vortical veil Σ. The selection of position of points on the surface of the wing in which the non-passage condition is satisfied is significant for proof of the convergence of the process with a decrease in the sizes of the coordinate grid cells. Selected as such points are points M(x, z) with coordinates $x = \xi_i + \frac{\Delta\xi_i}{2}$, $z = \zeta_j$, which lie in the geometric centers of the coordinate cells (see Fig. 1).

Subsequently, discussions connected with the limiting transition with a decrease in coordinate cell $\Delta\xi_i$, $2\Delta\zeta_j$ without loss of generality are conducted for a flat wing of arbitrary form in a plane. Everything which has been stated is correct for arbitrary smooth surfaces without lines of discontinuity of curvature. In the case of a flat wing, the orthogonal coordinate system on the wings is rectilinear and, accordingly, each Π-shaped vortex connected with the wing consists of three rectilinear segments.

The velocity induced by the elementary vortex $d\bar{l}$ of intensity Γ at some point which is at a distance r from the middle of the elementary vortex is determined from the Biot-Savart formula

$$d\bar{V} = \frac{\Gamma}{4\pi} \cdot \frac{[d\bar{l} \times \bar{r}]}{r^3}.$$

In accordance with this formula for points of a flat wing the velocities induced by the vortical lines lying in the same wing plane are directed along the normals to this plane. In this regard, the value of the velocity induced in point M(x, z) on a wing by a Π-shaped vortex of intensity $\Delta\Gamma_{ij}$ which passes through point $N(\xi_i, \zeta_j)$ and consisting of rectilinear segments of finite length $2\Delta\zeta_j$, $\xi_i - x_3(\zeta_j - \Delta\zeta_j)$, $\xi_i - x_3(\zeta_j + \Delta\zeta_j)$, where $\xi = x_3(\zeta)$ - the equation of the wing's trailing edge, can be presented in the form

$$\Delta V_{ij} = -\frac{\Delta\Gamma_{ij}}{4\pi}[F(x, z, \xi_i, \zeta_j + \Delta\zeta_j) - F(x, z, \xi_i, \zeta_j - \Delta\zeta_j)]$$
$$\text{for } ~~\text{для}~~ (\xi_i, \zeta_j \neq z), \qquad (1)$$

where

$$F(x, z, \xi, \zeta) = \frac{1}{z-\zeta}\left[\frac{\sqrt{(x-\xi)^2+(z-\zeta)^2}}{x-\xi} - \frac{[x-x_3(\zeta)]}{\sqrt{[x-x_3(\zeta)]^2+(z-\zeta)^2}}\right],$$
$$\Delta V_j^0 = -\frac{\Delta\Gamma_i}{4\pi}[F(x, z, \xi_i, z+e_j) - F(x, z, \xi_i, z-e_j)],$$
$$\text{for } ~~\text{для}~~ (\xi_i, \zeta_j = z), \qquad (2)$$

where

$$\Delta\Gamma_i = \Delta\Gamma_{ij}(\xi_i, z), \ \Delta\zeta_j = e_j.$$

Point M(x, z) is selected at the center of a coordinate cell so that always in the case of a finite number of discrete vortices $\xi_i \neq x$, and $\zeta_j = z$ only for one column of coordinate cells. The velocity induced at point M(x, z) by all discrete Π-shaped vortices connected with the wing S is the sum of the velocities induced by each of the Π-shaped vortices. If we conduct a summation in a fixed line, discarding here the terms which correspond to the

value $\zeta = z$, and then the summation for all the lines, the velocity at point (**x**, z) can be presented in the form

$$V = -\frac{1}{4\Pi}\left\{\sum_{m}\sum_{n}{}' \rho(\xi_i, \zeta_j)\frac{F(x, z, \xi_i, \zeta_j + \Delta\zeta_j) - F(x, z, \xi_i, \zeta_j - \Delta\zeta_j)}{2\Delta\zeta_j} \times \right.$$
$$\left. \times\, 2\Delta\zeta_j\Delta\xi_i + \sum_{m}\rho(\xi_i, z)\left[F(x, z, \xi_i, z + \varepsilon_j) - F(x, z, \xi_i, z - \varepsilon_j)\right]\Delta\xi_i\right\}, \qquad (3)$$

where the intensity $\Delta\Gamma_{ij}$ of a Π-shaped vortex which passes through point ξ_i, $\zeta_j(\xi_m, \zeta_n)$ presented in the form $\Delta\Gamma_{ij} = \rho(\xi_i, \zeta_j)\,\Delta\xi_i$, in which regard $\Delta\xi_i$ - the distance along the axis ξ from point ξ_i, $\zeta_j(\xi_m, \zeta_n)$ of this vortex to the point $\xi_i + \Delta\xi_i$, $\zeta_j(\xi_{m+1}, \zeta_n)$ through which the next Π-shaped vortex passes. In formula (3), summation for a column where $\zeta_n = z$ is excluded in the double sum; the summation for this column is segregated separately.

With an infinite increase in the number of discrete vortices, where first $2\Delta\zeta_j \to 0$, and then $\Delta\zeta_j \to 0$, the velocity induced in point (x, z) by all the joined vortices can be presented in the form

$$V = -\frac{1}{4\Pi}\left\{\iint_{S-2\varepsilon}\rho(\xi, \zeta)F'_{\zeta}(x, z, \xi, \zeta)\,d\zeta\, d\xi + \int_{x_{\text{п}}(z)}^{x_{\text{з}}(z)}\rho(\xi, z)\left[F(x, z, \xi, z + \varepsilon) - \right.\right.$$
$$\left.\left. - F(x, z, \xi, z - \varepsilon)\right]d\xi\right\}, \qquad (4)$$

where S - 2ε - the area of a wing with an excluded flat width 2ε near point $\zeta = z$; $\xi = x_{\text{з}}(\zeta)$, $\xi = x_{\text{п}}(\zeta)$ - the equation respectively of the trailing and leading edges of the wing, and the derivative of the function $F(x, z, \xi, \zeta)$ has the form

$$F'_{\zeta}(x, z, \xi, \zeta) = \frac{1}{(z-\zeta)^2}\left\{\frac{x-\xi}{\sqrt{(x-\xi)^2 + (z-\zeta)^2}} - \right.$$
$$\left. - \frac{[x - x_{\text{з}}(\zeta)]^3 + 2[x - x_{\text{з}}(\zeta)]^3 (z-\zeta)^2 - x'_{\text{з}}(\zeta)(z-\zeta)^3}{\{[x - x_{\text{з}}(\zeta)]^2 + (z-\zeta)^2\}^{3/2}}\right\}.$$

The function $F'_{\zeta}(x, z, \xi, \zeta)$ exists in the region $S - 2\varepsilon$. In the region $S - 2\varepsilon$ in the internal integral of a double integral of formula (4) integration by parts can be conducted. Assuming without loss of generality $z > 0$, we obtain

$$
\begin{aligned}
V = -\frac{1}{4\pi}\Bigg\{ & -\iint\limits_{S-2\varepsilon} \rho'_{\zeta}(\xi, \zeta) F(x, z, \xi, \zeta)\, d\zeta\, d\xi - \int\limits_{0}^{x_{\text{з}}(0)} \rho[\xi, z_{1}(\xi)] \times \\
& \times F[x, z, \xi, z_{1}(\xi)]\, d\xi + \int\limits_{0}^{x_{\text{з}}(0)} \rho(\xi, 0) F(x, z, \xi, 0)\, d\xi + \int\limits_{0}^{x_{\text{п}}(z-\varepsilon)} \rho[\xi, z_{\text{np}}(\xi)] \times \\
& \times F[x, z, \xi, z_{\text{np}}(\xi)]\, d\xi + \int\limits_{x_{\text{з}}(z-\varepsilon)}^{x_{\text{з}}(0)} \rho[\xi, z_{\text{np}}(\xi)] F[x, z, \xi, z_{\text{np}}(\xi)]\, d\xi - \\
& - \int\limits_{0}^{x_{\text{п}}(z-\varepsilon)} \rho(\xi, 0) F(x, z, \xi, 0)\, d\xi - \int\limits_{x_{\text{з}}(z-\varepsilon)}^{x_{\text{з}}(0)} \rho(\xi, 0) F(x, z, \xi, 0)\, d\xi + \\
& + \int\limits_{x_{\text{п}}(z-\varepsilon)}^{x_{\text{з}}(z-\varepsilon)} \rho(\xi, z - \varepsilon) F(x, z, \xi, z - \varepsilon)\, d\xi - \int\limits_{x_{\text{п}}(z+\varepsilon)}^{x_{\text{з}}(z+\varepsilon)} \rho(\xi, 0) F(x, z, \xi, 0)\, d\xi + \\
& + \int\limits_{x_{\text{п}}(z+\varepsilon)}^{x_{\text{з}}(z+\varepsilon)} \rho[\xi, z_{\text{np}}(\xi)] F[x, z, \xi, z_{\text{np}}(\xi)]\, d\xi - \int\limits_{x_{\text{п}}(z+\varepsilon)}^{x_{\text{з}}(z+\varepsilon)} \rho(\xi, z + \varepsilon) F(x, z, \xi, z + \varepsilon)\, d\xi + \\
& + \int\limits_{x_{\text{п}}(z)}^{x_{\text{з}}(z)} \rho(\xi, z)\,[F(x, z, \xi, z + \varepsilon) - F(x, z, \xi, z - \varepsilon)]\, d\xi \Bigg\}.
\end{aligned} \tag{5}
$$

For proof of the existence of a velocity on the wing which is determined by formula (5), it is necessary to make an assumption concerning the shape of the wing contour and concerning the nature of the vortical density on the wing. The continuity of equations $\xi = x_{\text{п}}(\zeta)$, $\xi = x_{\text{з}}(\zeta)$ of the leading and trailing edges of the contour at interval $a < \zeta < b$ (they may be lateral edges parallel to the axis ξ with $\zeta = a$, $\zeta = b$) is assumed for contour L. It is also assumed that on a wing, including the wing edge, the value of vortical density $\rho(\xi, \zeta)$ and the derivative $\rho'_{\zeta}(\xi, \zeta)$, through the value of which the limiting value of density of discrete vortical lines on wing S which coincide with the direction of the axis ξ is determined, satisfy the Helder condition.

We will demonstrate the existence of velocity on a wing, which is determined by formula (5), for the internal points of the wing. The double integral in formula (5) can be presented in the form

$$\iint\limits_{S-\sigma} \frac{\rho'_{\zeta}(\xi, \zeta)\,\Phi(x, z, \xi, \zeta)\,d\zeta\,d\xi}{(x-\xi)(z-\zeta)},$$

where expression

$$\Phi(x, z, \xi, \zeta) = \frac{\sqrt{(x-\xi)^2+(z-\zeta)^2}\,\sqrt{[x-x_{з}(\zeta)]^2+(z-\zeta)^2}-(x-\xi)[x-x_{з}(\zeta)]}{\sqrt{[x-x_{з}(\zeta)]^2+(z-\zeta)^2}} \quad (6)$$

is the continuous function for points of the wing (x, z) which do not lie on the trailing edge of the wing. This integral is the principal value of the repeated interval of the Cauchy type which exists for points which do not lay on the wing contour and, because of this, are internal points of each of the repeated intervals [5].

After the addition of simple integrals in which the expression $\rho(\xi, 0)\ F(x, z, \xi, 0)\ d\xi$ stand under the integral sign, two components remain which, on the basis of the mean value theorem, can be presented in the form

$$\int\limits_{x_{п}(z-\varepsilon)}^{x_{п}(z+\varepsilon)} \rho(\xi, 0)\,F(x, z, \xi, 0)\,d\xi + \int\limits_{x_{з}(z+\varepsilon)}^{x_{з}(z-\varepsilon)} \rho(\xi, 0)\,F(x, z, \xi, 0)\,d\xi =$$
$$= \rho[x_{п}(z), 0]\,F[x, z, x_{п}(z), 0]\,2\varepsilon + \rho[x_{з}(z), 0]\,F[x, z, x_{з}(z), 0]\,2\varepsilon,$$

where

$$F[x, z, x_{п,з}(z), 0] = \frac{1}{\varepsilon}\left\{\frac{\sqrt{[x-x_{п,з}(z)]^2+\varepsilon^2}}{x-x_{п,з}(z)} - \frac{[x-x_{з}(0)]}{\sqrt{[x-x_{з}(0)]^2+\varepsilon^2}}\right\}$$

- a function which is limited for points which do not lie on the wing edges (value z > 0). With the assumptions made above concerning the finiteness of the values of vortical density on the wing edges, the value of each of these components with $\varepsilon \to 0$ disappears.

The sum of the simple integrals which contain under the integral sign the function

$$F(x, z, \xi, z \pm \varepsilon) = \frac{1}{(\mp\varepsilon)}\left\{\frac{\sqrt{(x-\xi)^2+\varepsilon^2}}{x-\xi} - \frac{[x-x_{з}(z\pm\varepsilon)]}{\sqrt{[x-x_{з}(z\pm\varepsilon)]^2+\varepsilon^2}}\right\} =$$
$$= \frac{H(x, z, \xi, z\pm\varepsilon)}{(\mp\varepsilon)},$$

where

$$\lim_{\varepsilon \to 0} H(x, z, \xi, z \pm \varepsilon) = 0,$$

also disappears with $\varepsilon \to 0$. We will show this using as an example the function $F(x, z, \xi, z + \varepsilon)$:

$$\int_{x_{\text{II}}(z)}^{x_3(z)} \rho(\xi, z) F(x, z, \xi, z+\varepsilon)\, d\xi - \int_{x_{\text{II}}(z+\varepsilon)}^{x_3(z+\varepsilon)} \rho(\xi, z+\varepsilon) F(x, z, \xi, z+\varepsilon)\, d\xi =$$

$$= - \int_{x_{\text{II}}(z+\varepsilon)}^{x_3(z+\varepsilon)} [\rho(\xi, z+\varepsilon) - \rho(\xi, z)] F(x, z, \xi, z+\varepsilon)\, d\xi + \int_{x_{\text{II}}(z)}^{x_{\text{II}}(z+\varepsilon)} \rho(\xi, z) \times$$

$$\times F(x, z, \xi, z+\varepsilon)\, d\xi + \int_{x_3(z+\varepsilon)}^{x_3(z)} \rho(\xi, z) F(x, z, \xi, z+\varepsilon)\, d\xi = \int_{x_{\text{II}}(z+\varepsilon)}^{x_3(z+\varepsilon)} [\varepsilon \rho_z'(\xi, z) +$$

$$+ 0(\varepsilon^2)] \frac{H(x, z, \xi, z+\varepsilon)}{\varepsilon}\, d\xi - \frac{1}{\varepsilon} \int_{x_{\text{II}}(z)}^{x_{\text{II}}(z) + \varepsilon x_{\text{II}}'(z)} \rho(\xi, z) H(x, z, \xi, z+\varepsilon)\, d\xi -$$

$$- \frac{1}{\varepsilon} \int_{x_3(z) + \varepsilon x_3'(z)}^{x_3(z)} \rho(\xi, z) H(x, z, \xi, z+\varepsilon)\, d\xi.$$

In the first term the integrand expression, because of the property of the function H, disappears with $\varepsilon \to 0$ and the two following terms are presented in the form

$$-\rho\left[x_{\text{II}}(z) + \frac{\varepsilon}{2} x_{\text{II}}'(z), z\right] H\left[x, z, x_{\text{II}}(z) + \frac{\varepsilon}{2} x_{\text{II}}'(z), z+\varepsilon\right] x_{\text{II}}'(z) -$$

$$-\rho\left[x_3(z) + \frac{\varepsilon}{2} x_3'(z), z\right] H\left[x, z, x_3(z) + \frac{\varepsilon}{2} x_3'(z), z+\varepsilon\right] x_3'(z)$$

on the basis of the mean value theorem and, on the strength of the property of function H, also disappear with $\varepsilon \to 0$.

With $\varepsilon \to 0$, the remaining simple integrals in the right side of formula (5) are a contour interval

$$\oint_L \rho[\xi, f(\xi)] F[x, z, \xi, f(\xi)]\, d\xi,$$

where $\zeta = f(\xi)$ - the equation of the contour L, integration in terms of which is conducted in a counter clockwise direction. The contour integral can be written in the form

$$\oint_L \rho[\xi, f(\xi)] \frac{\Phi[x, z, \xi, f(\xi)]}{(x-\xi)[z-f(\xi)]} d\xi,$$

where the function Φ is determined by the relation (6). For points (x, z) which do not lie on loop L, two situations can be encountered with the condition of continuity of equations of the leading and trailing edges in the contour integral: 1) $x = \xi$, $z \neq f(\xi)$, 2) $x \neq \xi$, $z = f(\xi^0)$. The case where simultaneously $x = \xi$, $z = f(\xi)$, cannot exist for the interior points (x, z) of the wing.

In the first case the integral $\oint_L \frac{F_1(\xi)\,d\xi}{x-\xi}$, where $F_1(\xi) = \rho[\xi, f(\xi)] \cdot \frac{\Phi[x, z, \xi, f(\xi)]}{z-f(\xi)}$ - the continuous function, in which regard $F_1(x) = \rho[x, f(x)]$ is an integral of the Cauchy type and exists in the sense of the main value.

In the second case $\oint_L \frac{F_2(\xi)\,d\xi}{z-f(\xi)}$, where $F_2(\xi) = \rho[\xi, f(\xi)] \cdot \frac{\Phi[x, z, \xi, f(\xi)]}{x-\xi}$ the continuous function, in which regard $F_2(\xi^0) = 0$ for П-shaped vortices which end on the trailing edge, leads by the replacement of the variables $f(\xi) = t$ to a form of integral of the Cauchy type

$$\oint_L \frac{F_2[\xi(t)]\,dt}{f'[\xi(t)](z-t)},$$

where with $\xi \to \xi^0$ $t \to z$. This integral exists in the sense of the principal value with $f'(\xi^0) \neq 0$. For loop L with the condition of continuity of equations of the leading and trailing edges the value $f'(\xi) = 0$ may occur only for the end points of the contour $\zeta = a$, $\zeta = b$ which cannot be points $\xi = \xi^0$ for points (x, z) which do not belong to the contour L. If lateral edges occur which are parallel to the axis ξ with $\zeta = a$, $\zeta = b$, where $f'(\xi) = 0$, then the points of the lateral edges cannot be points $\xi = \xi^0$ for points (x, z) which do not belong to the wing contour either. This means that the contour integral exists in the sense of the principal value in the second case, too.

Thus, the velocity induced by the Π-shaped vortices located on the wing S with the limiting transition from the discrete scheme to the scheme of a vortical surface is determined for internal points of the wing by the formula

$$V=\frac{1}{4\pi}\left\{\iint_{S}\rho'_{\zeta}(\xi,\zeta)F(x,z,\xi,\zeta)\,d\zeta\,d\xi+\oint_{L}\rho[\xi,f(\xi)]F[x,z,\xi,f(\xi)]\,d\xi\right\}. \qquad (7)$$

The integrals which determined the velocity are integrals of the Cauchy type and, for internal wing points, exist in the sense of the principal value. Essential for the concept of velocity in the sense of the main value of Cauchy type integrals (including repeated intergrals) is the subdivision of the integration interval into parts in the vicinity of a special point: $0\leqslant\xi\leqslant x-\delta$, $x+\delta\leqslant\xi\leqslant x_0(0)$ and $a\leqslant\zeta\leqslant z-\varepsilon$, $z+\varepsilon\leqslant\zeta\leqslant b$. The selection of the position of point M(x, z) at the center of the coordinate grid which was done in the work for a discrete case insures the convergence of the integrals in the sense of the principal value with the limiting transition.

Finite (nonzero) values of the density of vortical lines on sections of the wing contour where the vortical veil does not descend provide infinite values of velocity within the framework of an ideal fluid. On the wing contour L the conditions for the existence of integrals through which the vortex-induced velocity is expressed will be satisfied in the case where the contour L represents a line which lies completely within the vortical surface S + δS. In other words, the vortical surface of the wing should continue continuously beyond the wing S. Then the contour L will become a line whose points are internal points of the vortical surface S + δS which lie within the contour L + δL. The existence of velocity in the sense of the principal value has been proven for internal points of the surface, in which regard the velocities at points of the contour L are determined by formula (7) where integration is conducted in terms of the area S + δS and in terms of the contour L + δL. The vortical veil which lies outside the

contour L + δL, the form of which in the nonlinear case is not known ahead of time, as has already been noted will provide a regular velocity component for the points of wing S and its edges L.

In the case under consideration, when the wing is a lifting surface without thickness, for the existence of finite velocities on the edges is it necessary to pose the condition of the smooth joining of the surface of the wing S and the vortical veil Σ decending from it and the condition of continuous transition of the vortical density of these surfaces on their boundary - the contour L. On contour L, satisfaction of the condition of velocity finiteness in the general case does not require the disappearance of the vortical density on the edge of the wing if the vortical veil does not descend from it. The disappearance of the density of vortical lines which coincide with the direction of the axis ξ on the trailing edge of the wing in the case of a linear scheme is caused by the type of vortical veil beyond the edge and follows from the conditions of descent from the wing edge formulated above. In a linear scheme, when the vortical lines beyond the edge have the direction of velocity at infinity, on the surface Σ there are no components of the vortical lines perpendicular to the axis ξ (the axis ξ on the wing is directed along the velocity at infinity) and from the condition of continuity of transition of the vortical surface S to the surface Σ it follows that on the trailing edge of the wing S the intensity of vortical density of the latter from the Π-shaped vortices should equal zero. For a vortical line (wing of infinite span) from the condition of finiteness of velocity we also obtain values of zero vortical density on the trailing edge of the vortical segment. In the case of an end point of a vortical line the condition of finiteness of velocity which is determined by an integral of the Cauchy type can be satisfied only with the disappearance of the vortical density at the end of the line [5]. In the case of descent of the vortices from the edge of a finite span wing the wing edges are not the end points of vortical lines and the vortical density on the line of descent in general does not equal zero.

The discrete scheme should incorporate that distribution of vortical lines which, with the limiting transition from a discrete scheme to a scheme of vortical surface, would ensure the continuity of transition of vortical density on the contour of the wing L. This will be ensured if the point which, in the discrete scheme, is considered the point of the wing edge will be limited from the direction of the vortical veil by a vortical line of the same intensity as from the interior side of the wing (Fig. 2). Since there are two directions of vortical lines on the wing S which are parallel to the axis ζ and axis ξ, here for the selected system of rectangular П-shaped vortices the limiting value of the density of discrete vortices on the wing in the direction of the axis ζ equals ρ (ξ = const, ζ) and the limiting value of the density of discrete vortices on the wing in the direction of the axis ξ equals $\int_{x_{\text{н}}(\zeta)}^{\xi} \rho'_{\zeta}(\xi, \zeta =$ const) $d\xi$, then for each point of the edge in which the conditions of nonpassage are satisfied two additional vortices are drawn in the general case. From the direction of the vortical veil one vortex is drawn parallel to the axis ξ and the other - parallel to the axis ζ.

The wing is divided by a discrete grid of coordinate lines into coordinate squares in the center of each of which, at point (m, n), the condition of nonpassage is satisfied. In this regard, on the sections of the contour which do not coincide with the direction of the coordinate lines the contour is replaced by a broken line located outside the wing (the area of the wing is taken with a surplus). The rectangular П-shaped vortical lines connected with the wing whose intensity is determined from the conditions of nonpassage are plotted on the left side of Fig. 2 by solid lines. The arrows indicate the selected positive direction. Only a portion of these lines which lie in one small square are plotted in the figure. In this regard, each of the connected vortical lines begins and ends on the trailing edge of the wing. The continuation of these vortical lines outside the wing, beginning with the trailing edge, coincides for direction with the velocity of the flow.

On the left side of the figure their continuation, now as free, is portrayed by solid lines which begin at the corresponding points of the trailing edge. The boundary points in which the conditions of nonpassage are satisfied and which, for a discrete scheme, are points of the contour are marked by small crosses. At these points, the conditions of descent must be satisfied, i.e., the conditions are incorporated which ensure the continuous transition of the vortical surface of the wing S to the vortical surface of the veil Σ.

For a trailing edge which is not parallel to the coordinate axis ζ, the boundary point is point (m, 1) which lies at the center of the rectangle $B_{m,1}F_{m,1}C_{m,1}D_{m,1}$, in which regard the sides $B_{m,1}D_{m,1}$ and $D_{m,1}C_{m,1}$ serve as the boundary of the wing S and the vortical veil Σ. To ensure continuity of the vortical surface in the direction of the axis ξ (vortical lines which coincide with the direction of axis ζ) along the side $D_{m,1}C_{m,1}$, the vortical line of intensity $\Delta\Gamma_{m,1}$ is introduced, where $\Delta\Gamma_{m,1}$ - the intensity of the П-shaped vortical line connected with the square whose center is point (m, 1). At points $D_{m,1}$ $C_{m,1}$ the vortical line undergoes a fracture and behaves like a free vortical line. To ensure the continuity of the vortical surface in the direction of the axis ζ along side $B_{m,1}D_{m,1}$ a vortical line is introduced with intensity $2\Delta\Gamma_{m,1} - \Delta\Gamma_{m,2}$. At points $B_{m,1}$, $D_{m,1}$ the vortical line which is introduced undergoes a fracture and subsequently behaves as a free vortical line. In Fig. 2 the vortices which are introduced are plotted by the broken lines while their selected positive direction is marked by arrows.

For a trailing edge which is parallel to the axis ζ, the boundary point is the point (m, n) which lies at the center of the rectangle $B_{m,n}F_{m,n}D_{m,n}C_{m,n}$, in which regard the side $B_{m,n}C_{m,n}$ serves as the boundary of the wing S and the vortical veil Σ. A vortical line of intensity $\Delta\Gamma_{m,n}$ is introduced to ensure the continuity of the vortical surface along the side $B_{m,n}C_{m,n}$. At points $B_{m,n}C_{m,n}$ the line which is introduced undergoes a fracture and

subsequently behaves as a free vortical line.

For lateral edges parallel to the axis ξ, the boundary point is the point (m, 1) which lies at the center of rectangle $B_{m,1} C_{m,1} D_{m,1} F_{m,1}$, in which regard side $B_{m,1} C_{m,1}$ serves as the boundary of the wing S and vortical veil Σ. Along the side $B_{m,1} C_{m,1}$ a vortical line of intensity $2\Delta\Gamma_{m,1} - \Delta\Gamma_{m,2}$ is introduced which continues to the end of the lateral edge where it descends as a free vortical line together with all the joined vortical lines which coincide with the lateral edge. At point $B_{m,1}$ the vortical line behaves like a free vortical line. The selected positive direction of the joined and introduced vortical lines is marked by the arrows.

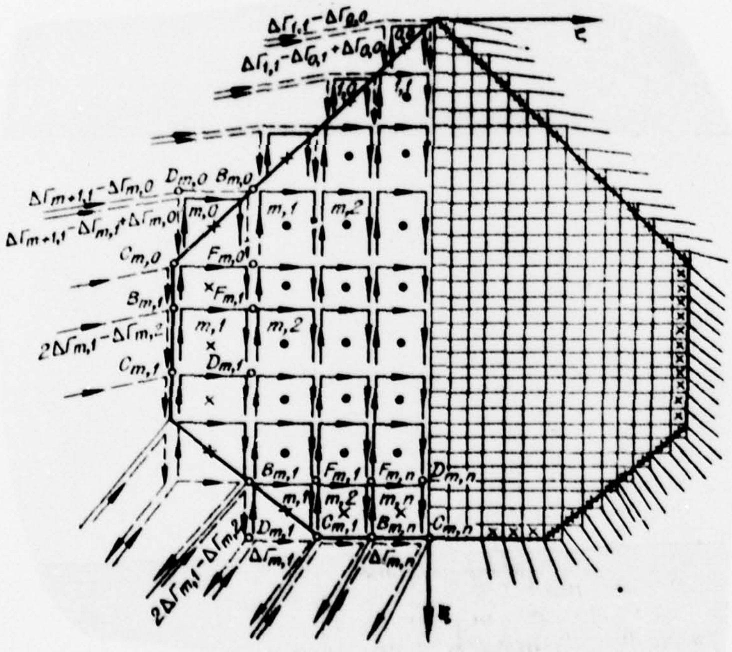

Fig. 2.

For a leading edge which is not parallel to the axis ζ, the boundary point is point (m, 0) which lies at the center of rectangle $D_{m,0}\ B_{m,0}\ F_{m,0}\ C_{m,0}$, in which regard sides $D_{m,0}B_{m,0}$ and $D_{m,0}C_{m,0}$ serve as the boundary of the wing S and the vortical veil Σ. A vortical line of intensity $\Delta\Gamma_{m+1,1} - \Delta\Gamma_{m,0} (\Delta\Gamma_{m+1,1} - \Delta\Gamma_{m,1} + \Delta\Gamma_{m,0})$, is

introduced along the side $D_{m,0}B_{m,0}$ ($D_{m,0}C_{m,0}$) and, from point $D_{m,0}$, behaves as a free vortical line and, from point $B_{m,0}$ ($C_{m,0}$) continues along the wing parallel to the axis ξ to the trailing edge where it descends as a free vortical line together with the corresponding joined vortices. The positive direction of the vortical lines which are introduced and joined is marked by arrows. The different signs with the value of intensity with coinciding arrows as well as the same signs with oppositely directed arrows signifies the mutual destruction of the action of vortical lines. As is evident from Fig. 2, the intensity of the vortical lines which are introduced and which ensure the continuity of transition of vortical surface S to surface Σ in the direction of the axis ξ and axis ζ for point (m, 0) are such that they mutually destroy the action of the joined vortex which is connected with point (m, 0) and the action of the components of the introduced vortices with intensity $\Delta\Gamma_{m,0}$. In the discrete scheme of the wing it is not necessary to introduce for consideration immediately the joined vortex with intensity $\Delta\Gamma_{m,0}$ which corresponds to point (m, 0) but to take the intensities of the introduced vortices as equal respectively to $\Delta\Gamma_{m+1,1}$ and $\Delta\Gamma_{m+1,1} - \Delta\Gamma'_{m,1}$.

Thus, the intensity of all vortices newly introduced on the edges of a wing and which ensure the finiteness of velocity at points of wing edges is expressed through the intensity of the previously constructed joined П-form vortices whose intensity is determined from the condition of nonpassage on the surface of the wing.

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